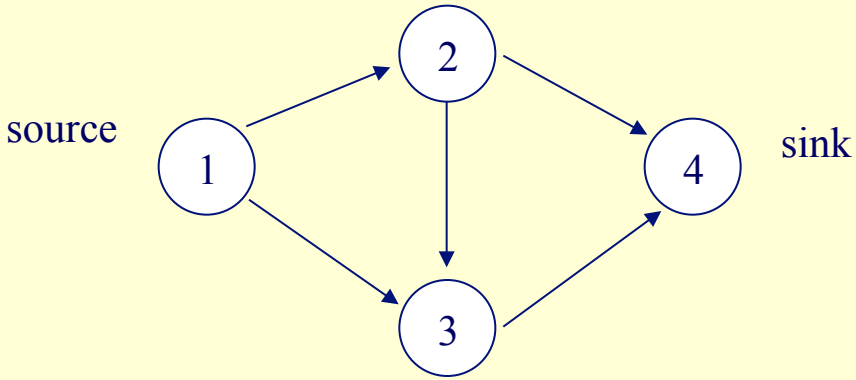
Linear Program Example

- Many problems involve “flowing”or moving some good or commodity over a network, some examples include:
  - Transportation network (vehicles)
  - Electricity (electrons)
  - Natural gas (methane molecules)
  - Coal distribution (tons of coal)
  - Project management project networks (project tasks)

Min Cost Flow Problem



let xij be the flow from
node i to node j

Nodes: {1,2,3,4}
Arcs: {(1,2),(1,3),(2,3),(2,4), (3,4)}

arc	unit cost	capacity
(1,2)	$100	5
(1,3)	$50	1
(2,3)	$75	2
(2,4)	$60	2
(3,4)	$90	2

Min Cost Flow Problem

minimize 100x12+50x13+75x23+60x24+90x34
s.t.

Node 1: x12+x13=3

Node 2: x12-x23-x24=0

Node 3: x13+x23-x34=0

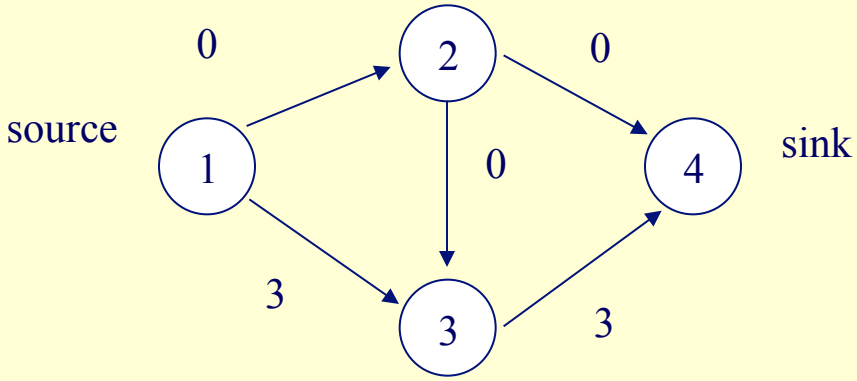
Node 4: x24+x34=3

x12,x13,x23,x24,x34 nonnegative

arc	unit cost	capacity
(1,2)	$100	5
(1,3)	$50	1
(2,3)	$75	2
(2,4)	$60	2
(3,4)	$90	2

Min Cost Flow Problem: Solution

Total cost =$420



Whoops! What's Wrong?

arc	unit cost	capacity
(1,2)	$100	5
(1,3)	$50	1
(2,3)	$75	2
(2,4)	$60	2
(3,4)	$90	2

Min Cost Flow Problem: Version 2

arc	unit cost	capacity
(1,2)	$100	5
(1,3)	$50	1
(2,3)	$75	2
(2,4)	$60	2
(3,4)	$90	2

minimize 100x12+50x13+75x23+60x24+90x34
s.t.

Node 1: x12+x13=3

Node 2: x12-x23-x24=0

Node 3: x13+x23-x34=0

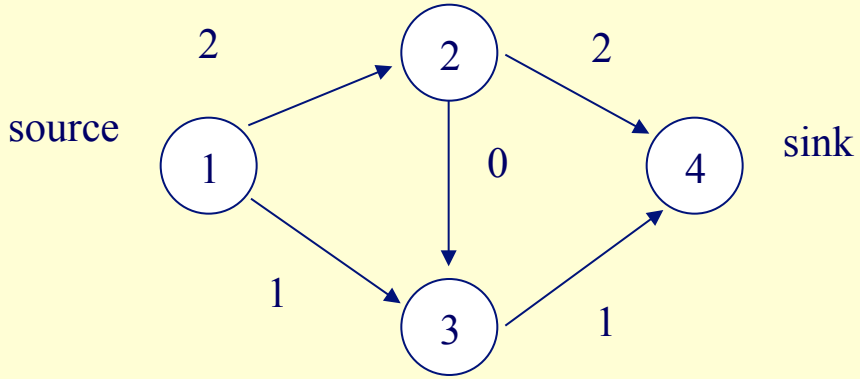
Node 4: x24+x34=3

x12<=5
x13<=1
x23<=2
x24<=2
x34<=2

Forgot the capacities!

x12,x13,x23,x24,x34 nonnegative

New solution with capacities



New total cost=$460, why should it be greater than previous cost of $420?